CONVERGENT SHOCK WAVE IN A HEAT CONDUCTING GAS

(SKHODIASHCHAIASIA UDARNAIA VOLNA V TEPLOPROVODNOM GAZE)

PMM Vol.29, № 2, 1965, pp. 334-336

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(Received October 19, 1964)

A convergent spherical shock wave intensifies without bounds, as it approaches the center. This was stated by Guderley [1], L.D. Landau and Staniukovich [2] using self-similar solutions of ordinary gas dynamics for wave focusing. However, in the corresponding physical phenomenon, the effect of heat transfer is considerable,,since the high temperatures involved result in radiant heat exchange.

It is of interest to consider wave focusing, taking into consideration the effect of radiant heat exchange, and in particular, to clarify whether in this case the energy accumulation also remains unbounded.

For simplicity, we carry this out only for sufficiently strong shock for which the radiation is everywhere in equilibrium with the medium, and the width of the precursor heating zone is large in comparison with the radiation path, including the early stage when this width is still small in comparison with the radius of the wave.

Heat transfer blurs the wave front: in front of the compression shock there appears a zone of precursor heating in which gas motion already occurs. The general properties of this phenomenon are described, for example, in the book by Zel'dovich and Raizer [3].

Let us calculate the width of the precursor heating zone firstly for a plane stationary shock ; for this, we write the conservation equations for a shock advancing in a cold gas

$$\rho u = \rho_0 D, \qquad p + \rho u^2 = \rho_0 D^2$$

$$\rho u \left(\frac{u^2}{2} + \frac{1}{\gamma - 1}\frac{p}{\rho}\right) + pu + Q = \frac{\rho_0 D^3}{2}$$

Here Q is the heat flux, and the other symbols are conventional (in the coordinates chosen, the shock is at rest).

Let us consider the case in which the radiant energy plays a role only in heat transfer, but its density $(4/c)\sigma T^4$ is still small in comparison with $p/(\gamma - 1)$, i.e. we shall use the equation of state of an ideal gas $p=(R/\mu)\rho T$, neglecting the radiation pressure.

For radiant heat transfer

$$Q = -\frac{lc}{3}\frac{d}{dx}\left(\frac{4}{c}\,\sigma T^4\right)$$

where c is the velocity of light, and l is the radiation path.

If the latter is given by Compton scattering, then

$$l \sim \rho^{-1} \quad \text{or} \quad l = L\,(\rho_0 / \rho) \qquad (L = \text{const})$$

Thus, we have four equations, using which u, p, ρ and dT/dx may all be expressed through T.

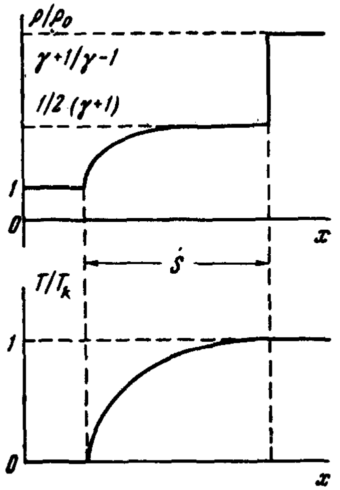

Fig. 1

Omitting the calculations, we give only the results for the quantities ρ and dT/dx

$$\frac{\rho}{\rho_0} = 2\left[1 + \left(1 - \frac{8(\gamma-1)}{(\gamma+1)^2}\tau\right)^{1/2}\right]^{-1}$$

$$\left(\tau = \frac{T}{T_k}, \quad T_k = \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{\mu D^2}{R}\right) \qquad (1)$$

Here T_k is the final temperature behind the wave;

$$\frac{d\tau}{dx} = \frac{1}{B\tau^3} \times$$

$$\times\left[\frac{2\tau/(\gamma+1) - 1}{1 + \sqrt{1 - 8\tau(\gamma-1)/(\gamma+1)^2}} + \frac{1}{2}\right]$$

$$B = \frac{256}{3}\frac{(\gamma-1)^4}{(\gamma+1)^8}\frac{L\mu^4\sigma D^5}{\rho_0 R^4}. \qquad (2)$$

In the precursor heating zone, τ varies from 0 to 1, while ρ/ρ_0 increases from 1 to the value $\frac{1}{2}(\gamma + 1)$ (as is evident from (1)) and then, by a jump, reaches its final value of $(\gamma + 1)/(\gamma - 1)$ $(\gamma \leqslant 3)$. Integrating (2) with respect to τ from 0 to 1, we find the complete width S of the heating zone in front of the compression shock

$$S = BJ, \quad J = \int_0^1 \frac{\left(1 + \sqrt{1 - 8\tau(\gamma-1)/(\gamma+1)^2}\right)\tau^3 d\tau}{2\tau/(\gamma+1) - 1 + {}^1/_2\left(1 + \sqrt{1 - 8\tau(\gamma-1)/(\gamma+1)^2}\right)}$$

For $\gamma = 1.4$,

$$J = 0.871, \qquad S = 1.73 \cdot 10^{-3}\,\frac{L\sigma\mu^4 D^5}{\rho_0 R^4} \qquad (3)$$

A diagram of the spacial distribution of ρ and T in the heating zone is shown in Fig.1.

Let us return to the convergent wave. As is evident from (3), the width of the heating zone increases rapidly with increase in D, i.e. as the wave approaches the center. The arrival of the heat wave at the center of the front is a characteristic moment of stopping of further temperature rise, and the temperature reached at that instant will be of the order of the maximum temperature of the entire process.

In a convergent wave $D = A / r^{\alpha}$ ($\alpha = 0.395$ for $\gamma = 1.4$), where A characterizes the strength of the shock (its velocity at unit radius).

The characteristic dimension of the wave r_0 is determined assuming that S is of the orser of r_0 . Substituting the expression for D in (3) and setting $S \sim r_0$, we obtain a relation for r_0 , from which we get

$$r_0 \sim \left(\frac{L\mu^4 \sigma A^5}{\rho_0 R^4} \right)^{\frac{1}{1+5\alpha}}$$

We determine the maximum temperature of the process. In a shock wave, $T \sim D^2/R$, i.e. in the convergent wave

$$T \sim \mu A^2 / R r^{2\alpha}$$

Substituting for r the value of r_0 , we get an expression for the maximum temperature

$$T_{\max} = \mathrm{const}\,(\rho_0^{2\alpha} R^{3\alpha-1} A^2 \mu^{1-3\alpha} L^{-2\alpha}\, \sigma^{-2\alpha})^{\frac{1}{1+5\alpha}}$$

In particular, for $\gamma = 1.4$ $(\alpha = 0.395)$

$$T_{\max} = \mathrm{const}\; \rho_0^{0.265} R^{0.060} \mu^{-0.060} A^{0.672} (L\sigma)^{-0.265}$$

Here const denotes a dimensionless coefficient of the order of unity. It can only be found from the complete solution of the problem of the wave focusing with heat conduction, which in principle can be done numerically.

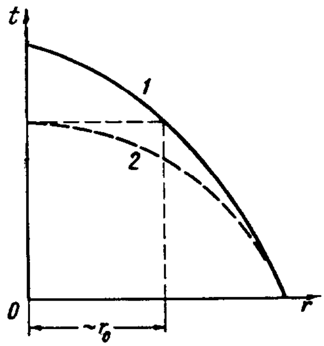

Fig. 2

Thus, in the presence of heat transfer, the temperature attained is finite, but with increasing wave strength (increasing A), this temperature can become arbitrarily high. In this sense, this temperature limitation by heat transfer is not a fundamental one.

A diagram of the appearance of the wave focusing in the present case is given in Fig.2.

The heat wave and the shock wave arrive successively at the center. Each of these near the center is described by its own self-similar solution (which we shall not consider), and a general self-similar solution for the entire process does not exist.

We shall consider the qualitative behavior of the shock wave at the center.

In the focusing stage, the temperature is constant, near the center it depends neither on r nor on t ; i.e. the wave is isothermal. Its amplitude may tend to zero, to a finite limit, or to infinity (oscillations are excluded). We shall show that the third possibility can be realized.

Tending to zero is excluded since it is well known that every weak shock before the center cannot weaken, but must increase in strength according to Formula

$$\Delta p \sim r^{-1}$$

If the amplitude would tend to a finite limit, then such a wave near the center would be described by a self-similar solution with constant amplitude.

Substituting such solution into the equations of motion, we shall see that the equations are not satisfied. Only the third case remains, i.e. unbounded increase (the law of this increase is not established). Thus, heat transfer only changes the unbounded accumulation but does not eliminate it : instead of finite density and infinite temperature, we obtain finite temperature and infinite density.

The author thanks Iu.P. Raizer for valuable remarks.

BIBLIOGRAPHY

1. Guderley, G., Starke kugelige und zylinrische Verdichtungsstösse in der Nähe des Kugelmittelpunktes bzw der Zylinderachse Luftfahrtforchung, 1942, v.19, №.9.

2. Staniukovich, K.P., Neustanovivshiesia dvizheniia sploshnoi sredy (Unsteady Motions of Continuous Media). Gostekhizdat, 1955.

3. Zel'dovich, Ia.B. and Raizer, Iu.P., Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh iavlenii (Physics of Shock Waves and High-temperature Hydrodynamic Phenomena). Fizmatgiz, 1963.

Translated by C.K.C.